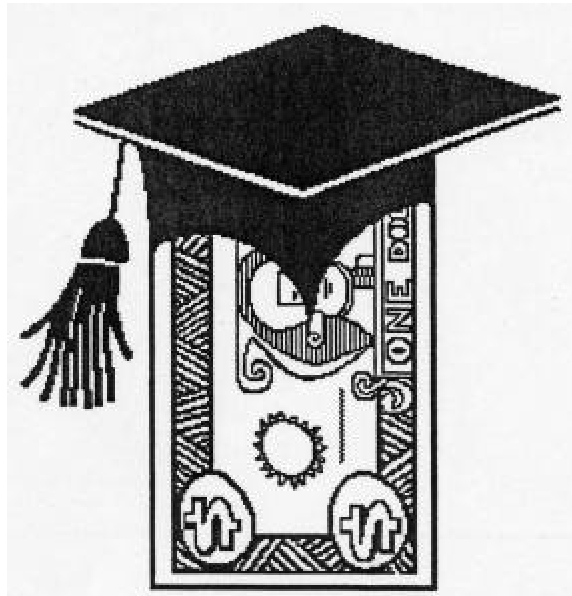
Hard Concepts



This section includes problems that range from difficult to very difficult math problems. They are grouped into units titled Concept #1 through Concept #10. These are the most difficult concepts found on the SAT/PSAT.

- This particular section was designed to raise the scores of those students who are already doing well on the SAT/PSAT to an even higher level.

- The problems in this section are representative of the ones on the SAT/PSAT that are extremely tricky. Detailed discussions of the concepts point out the traps set for students and show them how to identify and avoid those traps.

- Several practice problems are given for each concept in order to assist the student in gaining mastery of that type problem.

- A final section called The Big Review is included which covers all the ten concepts. Detailed explanations are given for solving each problem.

Note: We recommend that this section on hard concepts be given as a review to all students in higher level math courses before their testing date.

Math concept #1

A very difficult problem on a PSAT or SAT is one in which you must find among the five choices the correct expression to answer the question.

Example 1: The price of a dog, after it was reduced 20%, was D dollars. What was the price of the dog before the reduction?
(a) $1.80D (b) $1.25D (c) $1.20D (d) $0.80D (e) $0.75D

This type of problem can be solved by using algebra, but for most students it is very difficult to solve with algebra. If you can, more power to you.

The following is another method, called the *3-step E-Z McSqueezy Method*. This method allows you to use arithmetic to solve a difficult algebra problem.

Step 1. Choose your own value for any variables in the problem.
In this problem, choose $100 for the original price of the dog. It is usually good to use 100 in a percent question. After a 20% reduction, the price will be $80. Therefore, **D = 80**.

Step 2. Answer the question using your values for the variables.
The question asks for the price of the dog before the reduction. This is the original price or $100.
After answering their question with your values, put a big circle around your answer.

100

Step 3. Insert your values for the variables into each of the five answer choices and find the one that matches your answer.
In this problem, when you insert 80 into choice (b), you get $1.25(80) = $100.
(Note: If more than one of the five choices gives your answer, redo the problem using different values for the variables.)

For practice, let's do the problem over with a different value for D. I suggest that when you have a problem like this, you write down the variables and your values for them along with your answer to the problem.

1. Let the dog's original price be $20. After a 20% discount, the price will be $16. Therefore, **D = 16**.
2. Your answer to the question, the price of the dog before the reduction is $20.

20

3. You now insert 16 into each of the answer choices to find which will give your answer of 20. Choice (b) works. $1.25(16) = 20.

You are the champ if you can do example 2 below using algebra. See what you can do and then try the 3-step E-Z McSqueezy Method. A solution is below the dotted line.

Example 2: The sum of the first n positive integers is s. Which of the following equals the sum of the next n integers in terms of n and s?
(a) ns (b) $2n + s$ (c) $n^2 + s$ (d) $n + s$ (e) $n + 2s$

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1. I will let **n = 4**, and therefore, **s will equal 10**. $1 + 2 + 3 + 4 = 10$
2. My answer to the question, the sum of the next n (4) integers, is 26.
   $5 + 6 + 7 + 8 = 26$
3. I now substitute my values for n and s into the answer choices to find which matches my answer of 26. Choice (c) $4^2 + 10 = 26$, my answer.

Example 3: Joe took h hours to drive to Houston. How many hours would his trip have taken if he had increased his average speed by 10%?

(a) $\frac{11h}{10}$ (b) $\frac{10h}{11}$ (c) $\frac{h}{10}$ (d) $\frac{10h}{9}$ (e) $h - \frac{h}{10}$

To do this problem, you must know that distance = (rate)(time) or $d = rt$.

This means that the time = distance/rate or $t = \frac{d}{r}$ and rate = distance/time.

1. Let **h = 4** and the distance to Houston be 240 miles. This means that his average speed is 60 miles per hour.
2. If his speed increases by 10%, it will be 66 miles per hour and the time for the trip will be 240/66. This reduces to 40/11. This is my answer.
3. I now substitute my value for h into the answer choices to find which matches my answer of 40/11. Choice (b) 10(4)/11 = 40/11, my answer

Example 4: If $y = \dfrac{1}{1 + \frac{1}{x}}$, which of the following is equal to 2y?

(a) $\dfrac{2}{2 + \frac{2}{x}}$ (b) $\dfrac{2}{1 + \frac{2}{x}}$ (c) $\dfrac{1}{\frac{1}{2} + \frac{1}{2x}}$ (d) $\dfrac{1}{1 + \frac{1}{2x}}$

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1. Let **x = 1**.
2. Therefore, $y = 1/2$ and **2y = 1**. 1 is my answer.
3. I now substitute my value for x into the answer choices to find which matches my answer of 1. Choice (c)

Example 5: The total cost of g gallons of gas is d dollars. What is the cost of 2 gallons of gas? All answers are in dollars.

(a) $\frac{2d}{g}$ (b) $\frac{d}{2g}$ (c) 2gd (d) $\frac{2g}{d}$ (e) $\frac{gd}{3}$

1. Let **g = 2** and **d = 10**.
2. My answer to the question, the cost of 2 gallons of gas, is $10.
3. I now substitute my value for g and d into the answer choices to find which matches my answer of 10. Choice (a) 2(10)/2 = 10

Example 6: Water drips out of a bathtub faucet at a constant rate of 1 drop every n seconds. The bathtub fills at a rate of g gallons per hour. At these rates, how many drops are there in 1 gallon?

(a) $\frac{ng}{60}$ (b) $\frac{60}{ng}$ (c) $\frac{3600}{ng}$ (d) $\frac{60n}{g}$ (e) $\frac{3600n}{g}$

Example 7: A car travels m miles in t minutes. At the same rate, how many hours will it take the car to travel in 20 miles?

(a) $\frac{20t}{m}$ (b) $\frac{3m}{t}$ (c) $\frac{m}{20t}$ (d) $\frac{3t}{m}$ (e) $\frac{t}{3m}$

Example 8: The price of a cat, after it was increased by 25%, is D dollars. What was the price of the cat before the increase?
(a) $0.80D (b) $0.75D (c) $1.20D (d) $1.25D (e) $2.50D

Example 9: If $y = \frac{1}{1 + \frac{1}{x}}$, which of the following is equal to (1/2)y?

(a) $\frac{1}{2 + \frac{2}{x}}$ (b) $\frac{2}{1 + \frac{2}{x}}$ (c) $\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2x}}$ (d) $\frac{1}{2 + \frac{1}{x}}$

Example 10: The price of 5 pounds of apples is x dollars. The apples weigh an average of 1 pound for every 6 apples. Which of the following expressions is the average price, in cents, of a dozen such apples?
(a) $\frac{x}{40}$ (b) 10x (c) $\frac{100x}{3}$ (d) 40 x (e) $\frac{10x}{3}$

Example 11: EZ Auto Rental rents cars at the rate of d dollars per day and c cents per mile. A car was rented for 18 days and was driven 1,200 miles. What amount, in dollars, was owed?

(a) 30cd (b) $\frac{18(d + 66\ 2/3\ c)}{100}$ (c) 18(d + 66 2/3 c) (d) 6(2d + 3c)

(e) 6(3d + 2c)

Example 12: A train travels m miles in h hours and 20 minutes. What is the average speed of the train in miles per hour? (This one is probably easier to do with algebra, remembering that d = rt, but try to do it both ways.)

(a) $\frac{m + 20}{h}$ (b) $\frac{h}{m + 20}$ (c) $\frac{m}{h + \frac{1}{3}}$ (d) $m(h + \frac{1}{3})$ (e) $\frac{h + \frac{1}{3}}{m}$

Example 13: OPEC raised its oil price from x to (x + y). Which of the following expresses the fraction by which the price of oil was increased?

(a) $\frac{x}{x+y}$ (b) $\frac{x}{y}$ (c) $\frac{y}{1}$ (d) $\frac{y}{x}$ (e) $\frac{x+y}{y}$

Example 14: Joe took h hours to drive to Houston. How many hours would his trip have taken if he had increased his average speed by 20%?

(a) $\frac{5h}{6}$ (b) $\frac{6h}{5}$ (c) $\frac{h}{5}$ (d) $\frac{5h}{4}$ (e) $h - \frac{h}{5}$

Example 15: After Q ounces of oil and V ounces of vinegar were put into a jar to make some salad dressing, the jar was 1/3 full. Which expression expresses what fractional part of the jar was filled with oil?

(a) $\frac{1}{2Q + 2V}$ (b) $\frac{Q}{3QV}$ (c) $\frac{3Q}{Q + V}$ (d) $\frac{Q}{9(Q + V)}$ (e) $\frac{Q}{3Q + 3V}$

Answers: 1. b 2. c 3. b 4. c 5. a 6. c 7. e 8. a 9. a 10. d 11. e 12. c 13. d 14. a 15. e

% Math concept #2 %

Another difficult type of problem that might appear on the PSAT is one involving percents.

A difficult problem can be made a lot easier if you insert your own values for the variables in the problem.
In fact, if you ever see a variable, you should consider substituting in a value.

Example 1: If r is 25% of s and s is 40% of t, then r is what percent of t?
(a) 10% (b) 25% (c) 40% (d) 32.5% (e) 16%

Start by choosing values for r and s where r is 25% of s. I will choose <u>25</u> for r and <u>100</u> for s. Since s is now 100 and is 40% of t, then t would have to be 250. Now, since r is 25 and t is 250, r is 10% of t. So the correct answer is (a) 10%.

Of course, this problem may also be done with algebra.
Start with r = .25s and solving for s, you get 4r = s.
Also, s = .4t and substituting 4r for s, you get 4r = .4t.
Solve 4r = .4t for $\frac{r}{t}$ and you get $\frac{r}{t} = \frac{.4}{4}$, which is equal to 1/10 or 10%.

Unless you feel really comfortable doing these problems with algebra, I would suggest that you solve them by substituting your own values for the variables. If you have time, you might even do the problem by both methods to check your work.

Example 2: If r is 20% of s and s is 40% of t, then r is what percent of t?
(a) 10% (b) 80% (c) 20% (d) 8% (e) 30%

Example 3: The price of a car is reduced by 20% and the resulting price is then reduced by 40%. These two reductions would be the same as a single reduction of what percent?
(a) 60% (b) 30% (c) 52% (d) 48% (e) 80%
Do you see that 60% seems to be the correct answer? It's not.
Begin the problem by setting the original price of the car at $100.

Example 4: The price of oil rose 10% last year and from that point rose 30% this year. These two increases would be the same as a single increase of what percent?
(a) 40% (b) 20% (c) 57% (d) 41% (e) 43%

Example 5: The population of Richardson is 25% of the population of the rest of Dallas County. The population of Richardson is what percent of the entire population of Dallas County?
(a) 10% (b) 20% (c) 25% (d) 33 1/3% (e) 1/4%

A good start would be to call the population of Richardson 25 and the population of the rest of Dallas County 100. This would make the total population of Dallas County 125. Go from there.

Example 6: If x is 150% of y, then y is what percent of x?
(a) 50% (b) 33 1/3% (c) 66 2/3% (d) 75% (e) 100%

Example 7: Yesterday, a store sold apples at 6 pounds for a dollar. Today the store sells apples at 4 pounds for 50¢. What is the percent of the decrease in the price per pound?
(a) 4 1/6% (b) 12 1/2% (c) 16 2/3% (d) 20% (e) 25%

You just have to work out example 7. There are no variables for which you can choose your own value.

Example 8: In order to form a rectangle, the lengths of one pair of opposite sides of a square are increased by 20%, and the lengths of the other pair of opposite sides are decreased by 30%. The area of the rectangle is equal to what percent of the area of the square?
It always helps to draw diagrams.
(a) 16% (b) 90% (c) 50% (d) 84% (e) 60%

Example 9: During recent football tryouts, 50% of the players wanted to be the quarterback, 70% of those players trying out had never played before, and 40% of those that had never played before did not want to be the quarterback. What is the maximum percentage of the players who want to be the quarterback who have never played before?
(a) 28% (b) 30% (c) 42% (d) 50% (e) 84%
Hint: Have 100 players try out. Fifty want to be quarterback. What is the maximum number of these fifty who have never played before?

Example 10: A three-gallon mixture is 1/4 oil and 3/4 water. If one gallon of oil is added to the mixture, then approximately what percent of the four-gallon mixture is oil?
(a) 22% (b) 44% (c) 56% (d) 78% (e) 54%

There are no variables in this one, so you must work it out.

Answers 1. a 2. d 3. c 4. e 5. b 6. c 7. e 8. d 9. e 10. b

Avg. Math concept #3 Avg.

Another type of problem that can be very difficult is one involving averaging or an average.

The main thing to remember in doing one of these is to **FIND THE TOTAL AMOUNT OF STUFF**.

Example 1: A student answered 34 problems out of 35 on one test and 28 out of 50 on another test. How many problems must the student answer correctly on a third test of 35 questions in order to have 80% of the questions on all three tests correct?

In this problem, what is the total amount of stuff?
It would be the total number of questions, which is 120.
How many questions out of 120 must one answer correctly to score 80%?
It would be .80(120), which is equal to 96.
How many questions have already been answered correctly?
It would be 34 + 28, which is equal to 62.
How many questions still need to be answered correctly to get 96 right?
It would be 96 - 62, which is 34.

Example 2: A student answered 20 out of 30 questions correctly on one test and 35 out of 40 correctly on a second test. How many questions must the student answer correctly on a third test of 80 questions in order to have 66% of the questions on all three tests correct?
(a) 42 (b) 43 (c) 44 (d) 45 (e) 46

Example 3: Lolita's usual golf average is y. If she played six rounds of golf and had scores of y - 2, y + 4, y, y - 7, y + 5, and y - 3, what score must she make in a seventh game in order to have an average score of y - 1?
(a) y - 1 (b) y - 2 (c) y - 3 (d) y - 4 (e) y - 5

In example 3, $\frac{\text{the total amount of stuff}}{7} = y - 1$ (the average score)

Multiply both sides of this equation by 7, and
the total stuff will equal 7y -7. See if you can work it from there.

Example 4: On a 300-mile trip, a car went 30 miles per hour for the first 100 miles, 50 miles per hour for the second 100 miles, and 60 miles per hour for the third 100 miles. For what fractional part of the time taken for the entire 300-mile trip was the car going 30 miles per hour?
(a) 1/10 (b) 1/3 (c) 10/21 (d) 11/21 (e) 3/14

The total amount of stuff is the total time taken for the trip.

Example 5: The average weight of 3 children is 55 1/3 pounds. If each child weighs at least 54 pounds, what is the greatest possible weight of one of these children?
(a) 55 lbs. (b) 56 lbs. (c) 57 lbs. (d) 58 lbs. (e) 59 lbs.

Example 6: The average of three different integers is 17. The sum of the two greater integers is 49. What is the greatest possible value of any of the three integers?
(a) 25 (b) 46 (c) 48 (d) 17
(e) cannot be determined from the information given

Remember that there is very little chance that "cannot be determined from the information given" will ever be an answer to a difficult question.

Example 7: The average of five numbers is -7. The sum of three of the numbers is 9. What is the average of the other two numbers?
(a) 8 (b) 2 (c) -13 (d) -22 (e) -44

Example 8: The average age of Joe, Bill and Mary is 16. The average age of Joe and Bill is 13. The average age of Bill and Mary is 15. What is the average age of Joe and Mary?
(a) 16.5 (b) 14 (c) 48 (d) 16 (e) 20

Example 9: The average of three integers x, y and 8 is 10. Which of the following could not be the value of the product xy?
(a) 21 (b) 22 (c) 57 (d) 96 (e) 121

Example 10: (This is not one of the really difficult ones.)
If the average of z and 5z is 30, what is the value of z?
(a) 5 (b) 6 (c) 10 (d) 15 (e) 30

Example 11: If m is the average of x and 8, and n is the average of x and 4, what is the average of m and n?

(a) 6 (b) 12 (c) $\frac{x + 12}{2}$ (d) $\frac{x + 12}{4}$ (e) $\frac{x + 6}{2}$

Example 12: If y > 0 and m is the average of y, 2y and 30, and n is the average of y and 27, what is the relationship betwen m and n?
(a) m = n (b) m > n (c) m < n
(d) m could be equal to n, or m could be less than n, or m could be greater than n

<u>Try ex. 13 on your own and then look below at two ways to do the problem.</u>

Example 13: If q = 2r = 4t, then what is the average of q, r and t in terms of q?

(a) $\frac{7q}{3}$ (b) $\frac{13q}{7}$ (c) $\frac{7q}{8}$ (d) $\frac{7q}{6}$ (e) $\frac{7q}{12}$

<u>First way.</u> Figure out the total stuff (q + r + t) in terms of q.
We must find the average of q, r and t. Therefore, we must find r and t in terms of q. (r = q/2 and t = q/4)

Substitute into the expression q + r + t and get q + q/2 + q/4.
To average this, we must add the three terms up and then divide by 3.

Adding them, we get 7q/4. Dividing by 3, we get 7q/4 divided by 3.
After inverting the 3, we get 7q/4 • 1/3, which is equal to $\frac{7q}{12}$.

<u>Second way.</u> Assign values to q, r and t that will satisfy the equation q = 2r = 4t. How about 1 for t, 2 for r, and 4 for q? Your equation will be **4** = 2(**2**) = 4(**1**).
The average of q, r and t will be the average of 4, 2 and 1, which is 7/3.

Now substitute 4, our value for q, into each of the answer choices, and find which one matches 7/3. The choice that matches the 7/3 will be the correct answer.

If you substitute 4 in for q in choice (e), then you will get $\frac{7(4)}{12}$ which is $\frac{28}{12}$ which is $\frac{7}{3}$.

Example 14: Joe traveled ten miles to work at 40 miles per hour and traveled the same distance home at 60 miles per hour. What is his average speed?
(a) 50 mph (b) 42 mph (c) 45 mph (d) 48 mph (e) 52 mph

What is the total amount of stuff?
To find the average speed, you need the total distance divided by the total time. For instance, if you traveled 200 miles in 4 hours, your average speed would be 50 mph (200/4 = 50).

The total distance in this problem is 20 miles, 10 miles to work and 10 miles home. To get to work it takes 1/4 of an hour to drive the 10 miles at 40 mph. To get home it takes 1/6 of an hour to drive the 10 miles at 60 mph.
His total time driving is 1/4 + 1/6, which is 5/12 of an hour.
To find his average speed, divide the 20 miles (total distance) by the 5/12 (total time). You should get 48 mph.

If you are having trouble with this, you can still make an educated guess if you can answer this question. If a person drives 30 mph to a place and 40 mph returning over the same distance, is he going to spend more time driving 30 mph or 40 mph? Hopefully, you think 30 mph. If he spent the exact same time driving 30 mph as 40 mph, his average speed would be 35 mph. Since he is going 30 mph for a longer period of time than 40 mph, his average speed would be less than 35 mph. Using this logic in the above problem, you can determine that his average speed must be less than 50 mph, and you can eliminate answers (a) and (e).

Example 15: Joe traveled to work at 40 miles per hour and traveled the same distance home at 60 miles per hour. What is his average speed?
(a) 50 mph (b) 42 mph (c) 45 mph (d) 48 mph (e) 52 mph

Notice that this problem is just like example 14 except the distance to work is not given. Will this make a difference? No, since the distances going and returning are equal regardless of the total length.

Therefore, you need to make up any convenient distance and do the problem. A good distance would be 120 miles. It would take Joe 3 hours to get to work and 2 hours to get home, or a total of 5 hours for 240 miles. Notice that by dividing the total miles by the total time (240/5), you get 48 mph.

Example 16: Jenny spends a total of one hour driving to work and back. She averages 30 miles per hour going to work and 50 miles an hour returning. What is the total number of miles in a round trip?
(a) 30 (b) 31 1/2 (c) 37 1/2 (d) 40 (e) 50

What is your best guess? To work it, call the time going (t) and the time returing (1-t). Solve 30t = 50(1 - t) and go from there.

Answers 1. 34 2. c 3. d 4. c 5. d 6. b 7. d 8. e 9. b 10. c 11. e 12. d 13. e 14. d 15. d 16. c

Math concept #4

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

You can make some very difficult problems quite easy
if you make sure that you learn three common factors.

Notice that there are six expressions in the three equations. They are:

$(x + y)^2$	$(x - y)^2$	$(x + y)(x - y)$
$x^2 + 2xy + y^2$	$x^2 - 2xy + y^2$	$x^2 - y^2$

Whenever you see any of these six in any problem, <u>you should immediately write the expression that it is equal to.</u>

So, if you see $(x + y)^2$, you write $x^2 + 2xy + y^2$;

if you see $x^2 - y^2$, you write $(x + y)(x - y)$.

Example 1: If $x - y = 3$ and $x^2 - y^2 = 3$, what is the value of $x + y$?
(A) 0 (B) 1/2 (C) 1 (D) 1 1/2 (E) 2

I hope that after the lecture you started by writing $x^2 - y^2 = (x + y)(x - y)$.

Answers to the examples follow the last problem.

Example 2: $(x - y)^2 + 4xy = ?$

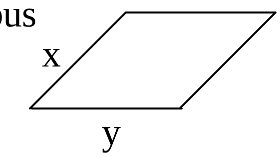
(a) $x^2 - y^2 + 4xy$ (b) $(x - 2y)^2$ (c) $(x - y)^2$ (d) $(x + y)^2$ (e) $(x + 2y)^2$

Example 3: If $x^2 - 9 = r$, $x + 3 = s$, and $rs \neq 0$, then $x - 3 = ?$

(a) $r + s$ (b) $r - s$ (c) $\frac{r+9}{s}$ (d) rs (e) $\frac{r}{s}$

Example 4: If $x - y = 11$ and $x^2 - y^2 = 165$, then $x = ?$
(a) 2 (b) 9 (c) 13 (d) 15 (e) 26

Example 5: For this rhombus $(x + y)(x - y) = ?$



(a) 0 (b) $2x$ (c) x^2 (d) $x^2 + 2xy + y^2$ (e) 1

Example 6: If $r = s + 1$ and $s = t^2 - 2t$, what is r in terms of t?

(a) $(t - 1)^2$ (b) $(t + 1)^2$ (c) $t^2 - t$ (d) $t + 1$ (e) $\frac{t^2 - 2t}{t+1}$

Example 7: $(25)^2 + 2(25)(75) + (75)^2 = ?$

(a) 6,550 (b) 10,000 (c) 400 (d) 9,900 (e) 3,950

Example 8: If $x + y = 2$ and $x - y = 6$, then what is $x^2 - y^2$?

(a) $\sqrt{12}$ (b) 20 (c) 40 (d) -32 (e) 12

Example 9: If $x + 2y = 6$ and $x - 2y = 8$, then what is $x^2 - 4y^2$?

(a) $4\sqrt{3}$ (b) 45 (c) 48 (d) 14 (e) 292

Example 10: If $x - y = 6$ and $x^2 - y^2 = 12$, then what is $x + y$?

(a) 1/2 (b) 2 (c) 18 (d) 72 (e) $6\sqrt{2}$

Example 11: If $(x + y)^2 = 100$ and $xy = 24$, then what is $x^2 + y^2$?
(a) 100 (b) 52 (c) 76 (d) 10 (e) 576

Example 12: If $(x - y)^2 = 16$ and $xy = 60$, what is $x^2 + y^2$?

(a) 92 (b) 32 (c) 66 (d) 76 (e) 136

Example 13: If $xy = 1/3$ and $x^2 + y^2 = 10/9$, what is $(x + y)^2$?

(a) 11/9 (b) 10/27 (c) 10/9 (d) 16/9 (e) 100/81

Example 14: If $x^2 + y^2 = 200$ and $xy = 28$, what is $(x - y)^2$?

(a) 0 (b) 172 (c) 144 (d) $10\sqrt{2} - 2\sqrt{7}$ (e) $(10\sqrt{2} - 2\sqrt{7})^2$

Example 15: (A tough one) If $a + 2b = x$ and $a - 2b = y$, find ab in terms of x and y.

(a) xy (b) $2xy$ (c) $\frac{x^2 - y^2}{4}$ (d) $\frac{x^2 - y^2}{8}$ (e) $\frac{x - y}{2}$

One way to start is to square both equations. This will result in:

$$a^2 + 4ab + 4b^2 = x^2 \quad \text{and} \quad a^2 - 4ab + 4b^2 = y^2$$

Now, subtract the second equation from the first. This will result in:

$$8ab = x^2 - y^2$$

Now, solve for ab and the answer is: $ab = \frac{x^2 - y^2}{8}$, which is choice (d).

Another way to solve the problem (and I think a lot easier) is to insert your own values for a and b, find x and y, find ab, and then substitute your values for x and y in the answer choices to see which choice gives you your ab. **Try this now.**

Example 16: If $(y + \frac{1}{y})^2 = 50$, then $\frac{1}{y^2} + y^2 = ?$

(a) 47 (b) 48 (c) 50 (d) 2498 (e) 2499

Example 17: $m + n = z$ and $m - n = 1/z$, assuming $z \neq 0$, $m^2 - n^2 = ?$

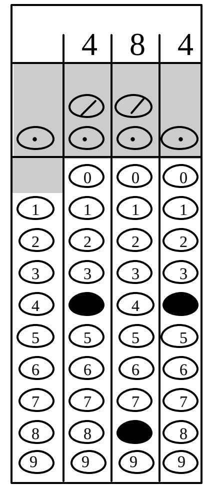
(a) 1 (b) 2 (c) z^2 (d) z (e) 0

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

18. If $xy = 105$, and $x^2 + y^2 = 274$, what is the value of $(x + y)^2$?

19. If $xy = 48$, $x^2 + y^2 = 100$, and $(x - y) > 0$, what is the value of $(x - y)$?

20. If $x + y = 10$, and $x - y = 5$, what is the value of $x^2 - y^2$?



21. If $x^2 - y^2 = 20$, and $x - y = 2$, what is the value of $x + y$?

22. If $x^2 - y^2 = 24$, and $x + y = 8$, what is the value of x?

23. If $x^2 + y^2 = 116$, and $xy = 40$, what is the value of $(x - y)^2$?

Review

Whenever you see any of these six expressions in a problem, immediately write the expression that it is equal to.

$(x + y)^2$	$(x - y)^2$	$(x + y)(x - y)$
$x^2 + 2xy + y^2$	$x^2 - 2xy + y^2$	$x^2 - y^2$

So, if you see $(x + y)^2$, you write $x^2 + 2xy + y^2$;

if you see $x^2 - y^2$, you write $(x + y)(x - y)$.

Answers 1. c 2. d 3. e 4. c 5. a 6. a 7. b 8. e 9. c 10. b 11. b 12. e 13. d 14.c 15. d 16. b 17. a 18. 484 19. 2 20. 50 21. 10 22. 11/2 or 5.5 23. 36

Math concept #5

$a^2 + b^2 = c^2$

3-4-5
5-12-13
8-15-17
7-24-25

in a 45-45-90 Δ, $c = a\sqrt{2}$

in a 30-60-90 Δ, side opposite $60° \angle = \dfrac{\text{hypotenuse } (\sqrt{3})}{2}$

You need to be able to apply the Pythagorean theorem and you should know 4 Pythagorean triples. Given one side, you also should be able to find the lengths of the other two sides of 45-45-90 triangles and 30-60-90 triangles.

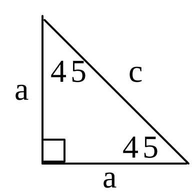
The Pythagorean theorem, $a^2 + b^2 = c^2$, means that the length of one leg of a right triangle squared plus the length of the other leg squared is equal to the length of the hypotenuse squared.

There are certain ratios, called Pythagorean triples, that will always work for right triangles and need to be memorized. These are:

3-4-5
5-12-13
8-15-17
7-24-25

In addition, there are certain relationships in 45-45-90 triangles and in 30-60-90 triangles that would be helpful to know.

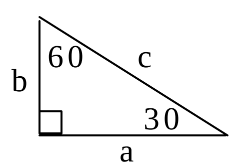
For a 45-45-90 triangle,



1. $\mathbf{a\sqrt{2} = c}$

2. $\mathbf{a = \dfrac{c\sqrt{2}}{2}}$

For a 30-60-90 triangle,



1. $\mathbf{a = \dfrac{c\sqrt{3}}{2}}$

2. $\mathbf{c = \dfrac{2a\sqrt{3}}{3}}$

3. $\mathbf{c = 2b}$

4. $\mathbf{b = \dfrac{c}{2}}$

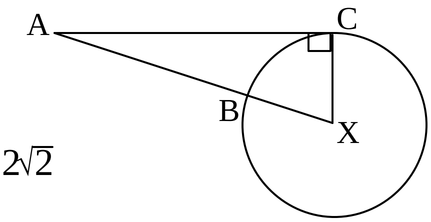
5. $\mathbf{a = b\sqrt{3}}$

6. $\mathbf{b = \dfrac{a\sqrt{3}}{3}}$

****Note****

It is very, very likely that you will have one or more problems that involve either the Pythagorean theorem or sides of these special triangles. You should definitely learn the triples and at least the three equations to the right that are circled.

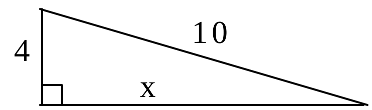
Example 1: In circle X, radius XC = 1 and AC = 2. Find AB.



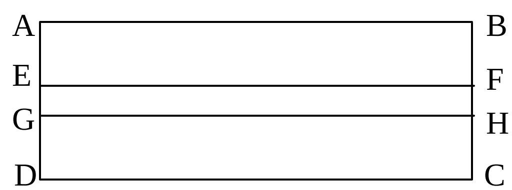
(a) 2 (b) 1 1/2 (c) $\sqrt{5}$ (d) $\sqrt{5} - 1$ (e) $2\sqrt{2}$

Example 2: In the following right triangle, find x.

(a) 6 (b) 8 (c) $2\sqrt{21}$ (d) $2\sqrt{29}$ (e) 9



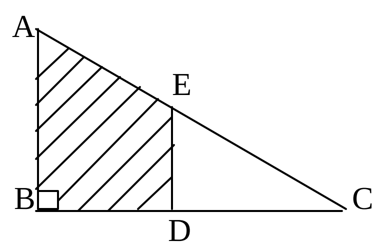
Example 3: ABCD is a rectangle. $\overline{EF} \parallel \overline{AB} \parallel \overline{GH} \parallel \overline{DC}$. The diagonal of the rectangle is 26. BF = 4, FH = 1 1/2, and HC = 4 1/2. Find the area of quadrilateral EFHG.



(a) 26 (b) 19 1/2 (c) 36 (d) 52 (e) 65

Example 4: D is the midpoint of $\overline{BC}$, and E is the midpoint of $\overline{AC}$. AB = 9 and EC = 7.5. What is the area of the shaded region? All answers are in square units.

(a) 18 (b) 27 (c) 36 (d) 40.5 (e) 67.5



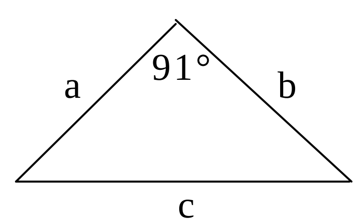
Example 5: A man is standing on a pier and is holding a rope that is attached to the front of a raft right at the water level. The rope is 25 feet long, and he is holding it 15 feet above the water level. If he pulls in 8 feet of rope, how many feet will the boat move? All answers are in feet.

(a) 8 (b) 10 (c) 12 (d) 14 (e) 15

Example 6: The legs of a right triangle are 6 and 8. The hypotenuse of the right triangle is also a leg of an isosceles right triangle. What is the length of the hypotenuse of the isosceles right triangle?

(a) 10 (b) 15 (c) $10\sqrt{2}$ (d) 20 (e) $15\sqrt{2}$

Example 7: Given the following diagram, which of the following statements is true?

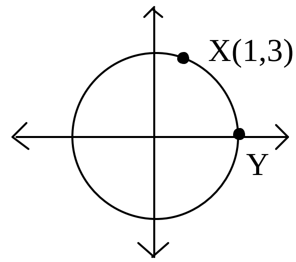


(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 < c^2$

(c) $a^2 + b^2 > c^2$ (d) $a + b = c$ (e) $a + b < c$

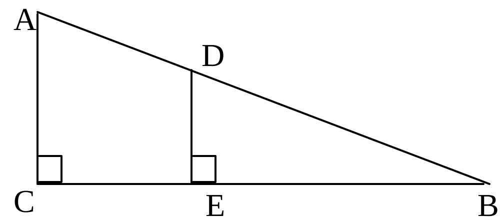
Example 8: Given the following diagram, what are the coordinates of point Y?

(a) (1,0) (b) (3,0)
(c) $(2\sqrt{2},0)$ (d) (2,0) (e) $(\sqrt{10},0)$



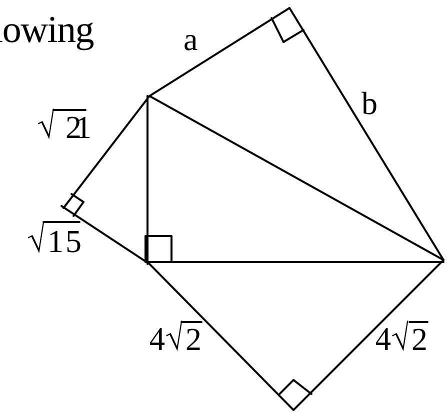
Example 9: In the given figure, CB =12, AC = 5, AD = 8x, and DB = 18x. What is the length of $\overline{DB}$?

(a) 7 (b) 8 (c) 9 (d) 11 (e) 13



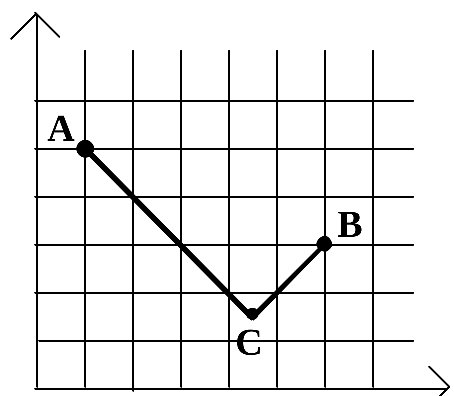
Example 10: Given the following diagram, find $a^2 + b^2$.

(a) 10 (b) 40 (c) 44
(d) 100 (e) 1360



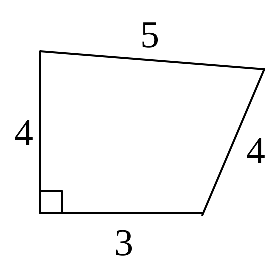
Example 11: In the given diagram, $m\angle ACB = 90°$. Find the value of $(AC)^2 + (BC)^2$.

(a) 6 (b) 25 (c) 49 (d) 35 (e) 29



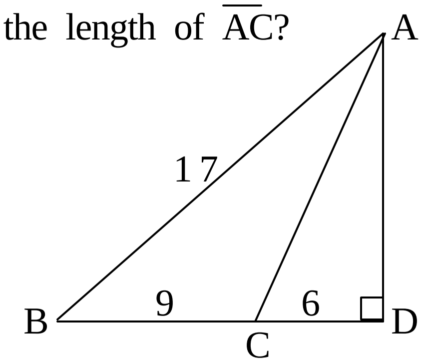
Example 12: What is the area of the given figure?

(a) 12 (b) $6 + 2\sqrt{21}$ (c) 16 (d) $6 + 4\sqrt{21}$
(e) It cannot be determined from the given information.



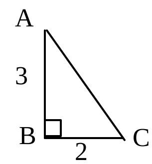
Example 13: In the given figure, what is the length of $\overline{AC}$?

(a) 7 (b) 7 1/2 (c) 8 (d) 9 (e) 10



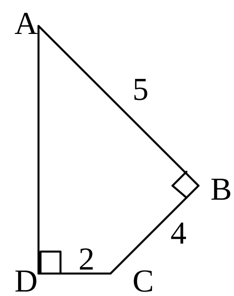
Example 14: What is the perimeter of ΔABC?

(a) 9 (b) 13 (c) $\sqrt{13}$ (d) $5 + \sqrt{13}$ (e) 8 1/2



Example 15: Given the following figure, what is the length of $\overline{AD}$?

(a) 5 (b) $\sqrt{13}$ (c) 6 (d) $\sqrt{37}$ (e) $3\sqrt{5}$

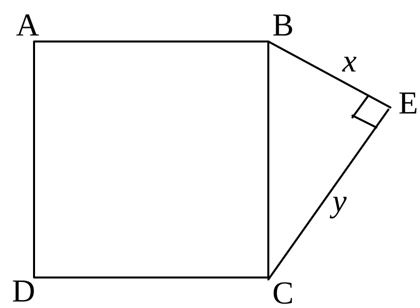


Example 16: In the given figure, quadrilateral ABCD is a square. Under which of the following conditions is the area of square ABCD equal to four times the area of right ΔBEC?

(a) if $x > y$ (b) if $x = y$ (c) if $x < y$

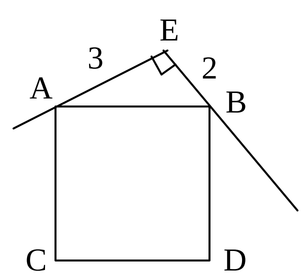
(d) if the perimeter of the square is equal to twice the perimeter of the triangle

(e) It is not possible for the area of the square to be equal to four times the area of the Δ.



Example 17: In the given figure, ABDC is a square, EB = 2 and AE = 3. What is the length of $\overline{ED}$?

(a) $4\sqrt{2}$ (b) $\sqrt{26}$ (c) $\sqrt{29}$ (d) $\sqrt{34}$ (e) 5



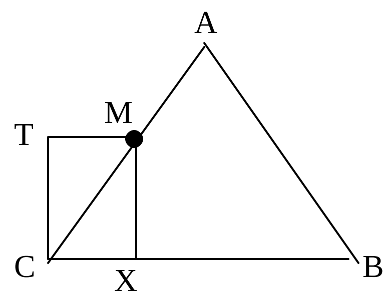
The following examples deal with the sides of 30-60-90 and 45-45-90 triangles.

Example 18: (a really tough one) A rectangular piece of paper is 24 inches long. What is the least possible width that the paper could have in order for one to cut out 11 circles, each with a radius of 2 inches? Choices are in inches.

(a) 8 (b) $2 + 2\sqrt{3}$ (c) 6 (d) $4 + 2\sqrt{3}$ (e) 7

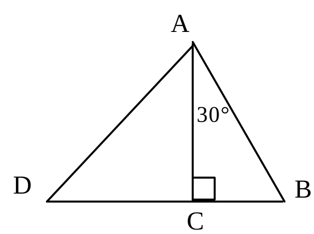
Example 19: ΔACB is an equilateral triangle. M is the midpoint of $\overline{AC}$. Quadrilateral TMXC is a rectangle. If AB = 4, then what is the area of ΔTMC?

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{2\sqrt{3}}{3}$ (e) $2\sqrt{3}$



Example 20: In the given figure, AB = 6, and AC = DC. Find the ratio of CB to DC.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$ (e) $\frac{1}{3}$



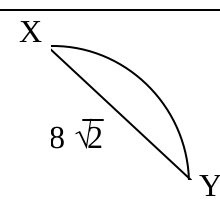
Example 21: Use the figure and the given information in example 20 to find the ratio of CB to AD.

(a) $\frac{1}{3}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{6}}{4}$ (d) $\frac{1}{6}$ (e) $\frac{\sqrt{6}}{6}$

Example 22: What is the area of the smallest equilateral triangle with sides whose lengths are whole numbers?

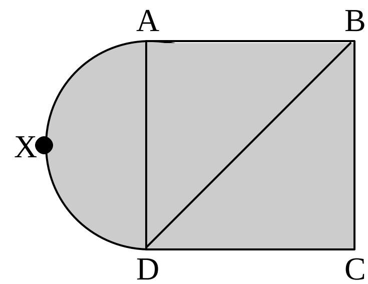
(a) $\frac{\sqrt{2}}{4}$ **(b)** $\frac{\sqrt{3}}{4}$ **(c)** $\frac{\sqrt{2}}{2}$ **(d)** $\frac{\sqrt{3}}{2}$ **(e)** $\sqrt{3}$

Example 23: Arc XY is 1/4 the circumference of a circle. If XY = $8\sqrt{2}$, what is the length of arc XY?



(a) $2\sqrt{2}\pi$ (b) $8\sqrt{2}\pi$ (c) 2π (d) 4π (e) 8π

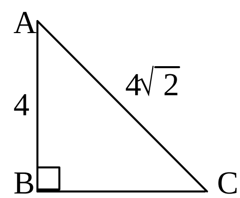
Example 24: In the given figure, quadrilateral ABCD is a square, and arc AXD is a semicircle. If BD = $\sqrt{2}$, what is the area of figure ABCDX?



(a) $\frac{1}{8}\pi + 1$ (b) $\frac{1}{4}\pi + 1$ (c) $\frac{1}{2}\pi + 4$

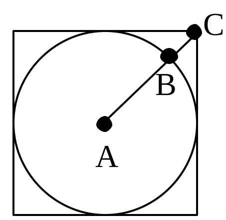
(d) $\frac{1}{2}\pi + 1$ (e) $\frac{1}{8}\pi + 4$

Example 25: In the given figure, which of the following statements is true?



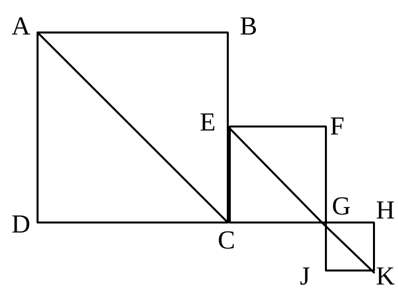
(a) $m\angle A > 45°$ (b) $m\angle A = 45°$ (c) $m\angle A < 45°$
(d) $m\angle A + m\angle C > m\angle B$ (e) No statements are true.

Example 26: Circle A is inscribed in a square that has a perimeter of 8. What is the length of $\overline{BC}$?



(a) 1 (b) $\frac{\sqrt{2}}{3}$ (c) $2\sqrt{2} - 2$ (d) $\frac{1}{2}$ (e) $\sqrt{2} - 1$

Example 27: In the given diagram, there are three squares and AB = 2EF = 4GH. What is the ratio $\dfrac{\text{length of } \overline{AC}}{\text{length of } \overline{EK}}$?



(a) $\frac{4}{3}$ (b) $\frac{4\sqrt{2}}{3}$ (c) $\frac{16}{9}$ (d) $\frac{2\sqrt{3}}{3}$ (e) $\frac{16}{5}$

Answers 1. d 2. c 3. c 4. d 5. c 6. c 7. b 8. e 9.c 10. d

11. e 12. b 13. e 14. d 15. d 16. b 17. c 18. d 19. b 20. d

21. e 22. b 23. d 24. a 25. b 26. e 27. a

Weird

abcdfghijklmnipqrt

Math concept #6

abcdfghijklmnipqrt

Symbols

A very tricky type of problem, until one is used to it, is one with an unfamiliar symbol. A little practice will make these quite a bit easier.

Example 1: Let **Y be defined for all Y by the equation **Y = 3Y +1. For example, **6 will equal 3(6) + 1, which is 19. If **3 - **Y = Y, what is Y?

(a) 2 (b) $2\frac{1}{4}$ (c) $2\frac{1}{2}$ (d) $2\frac{3}{4}$ (e) 3

Since **Y = 3Y + 1, you should begin the problem by saying, "I need to multiply 3 times whatever follows the ** and add 1." It helps if you formulate a sentence that shows you the meaning of the symbol.

Example 2: ΔΔX signifies the smallest integer greater than or equal to X. For example, ΔΔ4.1 = 5 and ΔΔ-2 = -2. What is ΔΔ-4.3 + ΔΔ4.3?
(a) -1 (b) 0 (c) 1 (d) 8 (e) 9

Example 3: Let X¶Y be defined as $\left(\frac{X}{Y}\right)^2$ for all nonzero numbers X and Y. What is 3¶B - (3¶ - B)?

(a) 6B (b) $\frac{18}{B^2}$ (c) $\frac{18}{B^4}$ (d) -1 (e) 0

Example 4: For all real numbers, (a, b, c, d)q(k, l, m, n) is equal to ak + bl + cm + dn. What is $\frac{(2, 2, 5, 3)q(3, -2, 4, 6)}{(-2, -2, -3, 3)q(5, -3, 4, 2)}$?
(a) -40 (b) 10 (c) 4 (d) -4 (e) 0

Example 5: If X is a positive even number, let X@ equal the sum of all positive even numbers from 2 to N. For example,
10@ = 2 + 4 + 6 + 8 + 10 = 30. Which of the following statements is false?
(a) X@ is always even.
(b) If X is divisible by 6, then X@ is divisible by 6.
(c) (X+2)@ - X@ = X + 2.
(d) If X is divisible by 4, then X@ is divisible by 4.

Example 6: The operation ® is defined on ordered pairs of numbers in the following way: (a,b)®(c,d) = (ac + bd, ad + bc).
If (a,b)®(m,n) = (a + b, a + b), what is (m,n)?

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,-1) (e) (1,1)

Example 7: For all numbers X, *X = X(X-1)(X-2). What is $\frac{*10}{*6}$?
(Be careful.)

(a) *3 (b) *4 (c) *6 (d) *8 (e) *1

Example 8: An operation Ω is defined for all real numbers r and s, and the equation r Ω s = rs^3. For which of the following is r Ω s = s Ω r?

I. r = 0, s = 1
II. r = s
III. r = -s

(a) I only (b) I and II only (c) II only (d) II and III only (e) I, II and III

Example 9: For all real numbers X and Y except zero, X !$!Y = $\frac{X^2}{Y^2}$.
If AB $\neq$ 0, which of the following is (are) always true?

I. X !$! Y = Y !$! X
II. (X !$! Y)(A !$! B) = (AX) !$! (YB)
III. (X !$! Y) ($\frac{-1}{X}$!$! $\frac{1}{Y}$) = -1

(a) I only (b) II only (c) III only (d) II and III only (e) I, II and III

Answers 1. b 2. c 3. e 4. d 5. d 6. e 7. a 8. e 9. b

Math concept #7

You will have to do some very tricky math problems that many students miss due to carelessness or because the problems defy your natural intuition.

A good example:
Which is greater, x^2 or x?
It seems that it would be x^2, but what if x is 1 or zero or a fraction?

Therefore, be careful!

Example 1: If y is an even number, what is the sum of the next two even numbers greater than $3y$?
(a) $6y + 2$ (b) $6y + 4$ (c) $6y + 6$ (d) $6y + 8$ (e) $6y + 10$

Example 2: If $\frac{A}{B} = \frac{3}{5}$ and $\frac{B}{C} = \frac{-5}{3}$, which of the following is (are) true?

I. $AB = 15$
II. $A/C = -1$
III. $(A + C)^2 = 0$

(a) none (b) I only (c) II only (d) II and III only (e) I, II and III

Example 3: Eight girls went to the movies together. Every ten minutes a pair of the girls would go to the snack bar together. If no two girls ever made more than one trip to the snack bar together, and trips to the snack bar were only made in pairs, what is the maximum number of trips that the group could send to the snack bar during the movie?

(a) 7 (b) 8 (c) 28 (d) 35 (e) 56

Example 4: If $5 = x^m$, then what will $5x$ equal?

(a) x^{m+5} (b) x^{m+3} (c) x^{m+1} (d) x^{2m} (e) $5x^m$

Example 5: X is the set of 10 consecutive odd integers whose sum is 20. Y is the set of 6 consecutive odd integers whose sum is 48. How many elements of X are also elements of Y?

(a) none (b) 1 (c) 2 (d) 5 (e) 6

Example 6: If $X^8 = 5$ and $X^7 = \frac{4}{T}$, what is X in terms of T?

(a) $5 - \frac{4}{T}$ (b) $\frac{4T}{3}$ (c) $\frac{4T}{5}$ (d) $\frac{1}{T}$ (e) $\frac{5T}{4}$

Example 7: If $(2^7 - 2^6)(2^5 - 2^4) = 2^n$, what is n?
(Don't work it out the long way.)

(a) 2 (b) 4 (c) 6 (d) 8 (e) 10

Example 8:

$$(1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32) - \frac{(1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32)}{2} =$$

(a) 3/4 (b) 7/8 (c) 15/16 (d) 31/32 (e) 63/64

Example 9: A pet store sells only dogs and cats. There are twice as many cats as dogs in the store. If 1/8 of the cats are males and 1/6 of the dogs are males, what fraction of the animals are males?
(Making your own problem with your own values is probably easier than solving with algebra.)

(a) 1/48 (b) 7/24 (c) 5/36 (d) 7/36 (e) 5/48

Example 10: A hockey team is going to play three games. It will get 10 points for a win, 3 points for a tie, and 0 points for a loss. How many different point possibilities are there that the team can have after completing the three games? (For example, if the team could end up with only 30 points, 9 points or 0 points, then there would be 3 possibilities.)

(a) 3 (b) 7 (c) 9 (d) 10 (e) 30

Example 11: If two odd numbers are not multiples of each other, which of the following must always be true?
(a) The sum of the integers is odd.
(b) Twice the smaller is greater than the larger.
(c) The smaller will divide evenly into three times the larger.
(d) The product cannot be negative.
(e) The product is an odd integer.

It is a wise person who knows the difference between the next two problems, and, on top of that, can do them both correctly.

Example 12: After Joe completed high school, he had made 3 A's for every 4 B's and 6 C's for every 5 B's. What was his ratio of C's to A's?

(a) $\frac{2}{1}$ (b) $\frac{5}{8}$ (c) $\frac{5}{3}$ (d) $\frac{8}{5}$ (e) $\frac{72}{5}$

Example 13: Emmitt gains as much yardage in three carries as Barry does in four. Barry gains as much in five carries as Thurman does in six. What is the ratio of Thurman's yards per carry to Emmitt's yards per carry?

(a) $\frac{1}{2}$ (b) $\frac{8}{5}$ (c) $\frac{3}{5}$ (d) $\frac{5}{8}$ (e) $\frac{5}{72}$

Example 14: There are eight men in a tennis league. If each man plays two matches against every other man, how many total matches are played in the league?

(a) 15 (b) 16 (c) 49 (d) 56 (e) 64

Example 15: If X and Y are positive integers and Y > X, which of the following statements is always true about the value of the following expression?

$$\left(\frac{X}{Y}\right)^2 - \sqrt{\frac{X}{Y}}$$

(a) The value is always zero.
(b) The value is always greater than zero.
(c) The value is always less than zero.
(d) The value is sometimes positive and sometimes negative.
(e) The value is always undefined.

Example 16: There are 75 girls in a row. The first three have red ribbons in their hair, and the next two have blue ribbons in their hair. The next three have red, and the next two have blue. This pattern continues all of the way down the row. What color ribbon do girls 68, 69 and 70 have in their hair?

(a) red, red, red (b) red, red, blue (c) red, blue, blue
(d) blue, blue, red (e) blue, red, red

Example 17: In all except one of the following, at least one of the coordinates of the ordered pair, when squared, is equal to the reciprocal of the other coordinate. For which ordered pair does this not hold true?

(a) (1,1) (b) $(\frac{1}{2},4)$ (c) $(\frac{1}{4},2)$ (d) $(\frac{1}{9},9)$ (e) (1,-1)

Example 18: In a junior high school with seventh and eighth grades, there are the same number of girls as boys. The eighth grade has 180 students, and there are 4 boys for every 5 girls. In the seventh grade there are 4 boys for every 3 girls. How many girls are in the seventh grade?
(a) 320 (b) 100 (c) 70 (d) 60 (e) 80

Example 19: X and Y are positive integers and X > Y. Which of the following must be less than $\frac{X}{Y}$?

(a) $\frac{X+1}{Y}$ (b) $\frac{3X}{3Y}$ (c) $\frac{X-0.1}{Y-0.1}$ (d) $(\frac{X}{Y})^2$ (e) $\frac{X+1}{Y+1}$

Example 20: What fraction of the number of integer multiples of 2 between 1 and 49 are also integer multiples of 3?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{16}{25}$ (d) $\frac{8}{25}$ (e) $\frac{8}{49}$

Example 21: What is the remainder when 256^{888} is divided by 10?

(a) 1 (b) 2 (c) 4 (d) 6 (e) 8

Example 22: If it takes 5 people 9 hours to do a certain job, how many hours would it take 2 people to do 2/3 of the job?

(a)15 hours (b) 2 2/5 hours (c) 22 1/2 hours (d) 3 3/5 hours (e)12 hours

This problem is fairly easy if you will remember one thing.

Start by figuring out how long it will take one person to do the job. You do this by figuring the total hours spent on the job. In this problem, five people each are working for 9 hours. That would mean a total of 45 (5 x 9) hours would be needed to finish the job.

So, if one person were doing it all by himself, he would need 45 hours. From there you figure how many hours it will take two people to do the job. This would be 22 1/2 hours.
Since these two people are just doing 2/3 of the job, it will take them 2/3 of 22 1/2 hours, which is 15 hours.

Example 23 is similar to Example 22.

Example 23: If it takes 9 people 10 hours to do a certain job, how long will it take 6 people to do 1/5 of the same job?

(a) 2 hours (b) 3 hours (c) 2/3 hour (d) 2 2/5 hours (e) 15 hours

Example 24: A certain basketball, after being dropped, will bounce 3/4 of its previous height. If the height after the 4th bounce is 2 1/4 feet, what was the height after the 1st bounce?

(a) 5 1/3 feet (b) 4 feet (c) 1 17/64 feet (d) 3 feet (e) 2 13/16 feet

Example 25: The sum of three consecutive even integers is M. In terms of M, what is the sum of the three consecutive integers that follow the greatest of the original even integers?

(a) M + 18 (b) M + 3 (c) M + 6 (d) M + 12 (e) M + 15

Example 26: If $\frac{A}{B} = \frac{2}{3}$ and $\frac{B}{C} = \frac{2}{5}$ and if A + B + C = 100, then what is A?

(a) 16 (b) 17 (c) 20 (d) 24 (e) 26 2/3

Example 27: If $\frac{9N}{4}$ is an odd integer, then which of the following could N not be equal to?

(a) -44 (b) 4 (c) 20 (d) 92 (e) 42

Example 28: The ratio of Joe's weight to Bill's is 2 to 3. The ratio of Joe's weight to Pete's is 3 to 4. What is the ratio of Bill's weight to Pete's weight?

(a) 9 to 8 (b) 8 to 9 (c) 1 to 2 (d) 2 to 1 (e) 3 to 4

An easy way to do #28 (a difficult problem) is to assign actual weights that fit the ratios.
Start by calling Joe's weight 30 and then find Bill's weight and Pete's weight by using the ratios. After that, compare Bill's weight to Pete's weight.

Example 29: $\frac{1}{10^{14}} - \frac{1}{10^{15}}$ is equal to which of the following?

(a) $-\frac{1}{10}$ (b) $\frac{1}{10^{15}}$ (c) $\frac{-9}{10^{15}}$ (d) $\frac{9}{10^{15}}$ (e) 0

Start #29, and any like it, by factoring. $\frac{1}{10^{14}}$ (? - ?)

Example 30: If $x = \sqrt{12} + \sqrt{48}$, what is the value of x^2?

(a) 60 (b) 54 (c) 96 (d) 120 (e) 108

Example 31: Given the set of integers that are greater than 0 and less than 100, how many of the integers are multiples of both 2 and 5?

(a) 19 (b) 10 (c) 9 (d) 48 (e) 58

Answers 1. c 2. d 3. c 4. c 5. d 6. e 7. e 8. e 9. c 10. d 11. e 12. d
13. d 14. d 15. c 16. c 17. d 18. d 19. e 20. b 21. d 22. a 23. b 24. a
25.d 26. a 27. e 28. a 29. d 30. e 31. c

<u>Math concept #8</u>

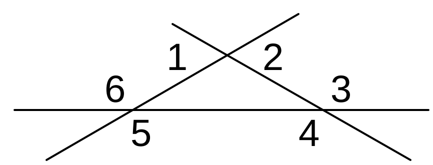
Difficult geometry problems

If you are not convinced that geometry teaches you how to think, or if you do not think it helps you in figuring out how much paint or fertilizer to buy, or if you feel that the only reason for geometry to exist is to give poor geometry teachers a job, think again. The difficult geometry problems are what separate the superstars from the others on the PSAT and SAT.

<u>Angle measure</u>

Example 1: What is the sum of the degree measures of the numbered angles?

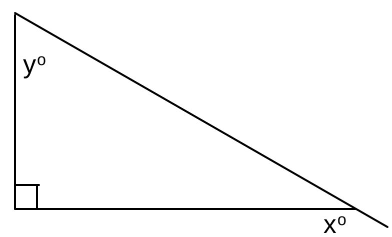
(a) 360° (b) 480° (c) 540°
(d) 720° (e) 900°



Example 2: In the given figure, what is y in terms of x?

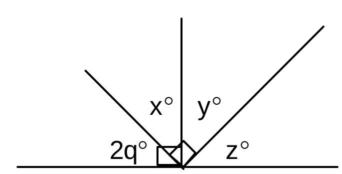
(a) $y = x - 90$ (b) $y = x$ (c) $y = 90 - x$

(d) $y = \frac{1}{2}x$ (e) $y = 180 - x$



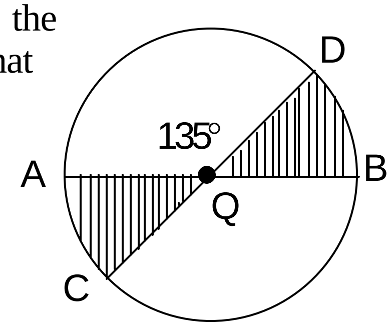
Example 3: Which of the following is always equal to $90 - q$?

(a) $x + z$ (b) $y + q$ (c) $z + q$ (d) q (e) $2q$

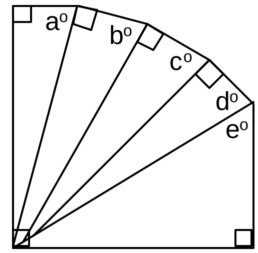


Example 4: The given circle has a center Q. If the radius of the circle is 4 and $m\angle AQD = 135°$, what is the total area of the shaded region?

(a) 2π (b) 4π (c) 6π (d) 8π (e) 12π



Example 5: In the given figure, what is the value of a + b + c + d + e?

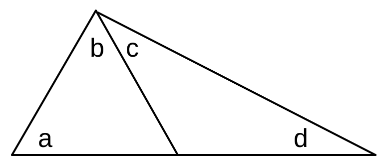


(a) 180 (b) 270 (c) 360 (d) 400 (e) 450

Example 6: Which of the following cannot be the measure of 1 exterior angle of a regular polygon?

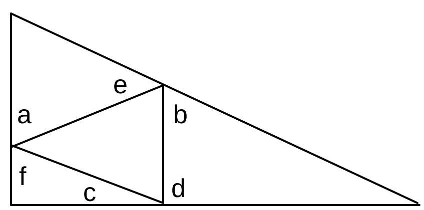
(a) 2 (b) 20 (c) 25 (d) 51 3/7 (e) 120

Example 7: In the given figure, what is the sum of the measures of a, b, c and d?



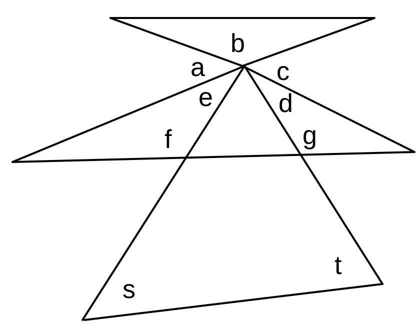
(a) 90 (b) 120 (c) 180 (d) 360
(e) It cannot be determined from the given information.

Example 8: In the given figure, what is the sum of the measures of a, b, c, d, e and f?



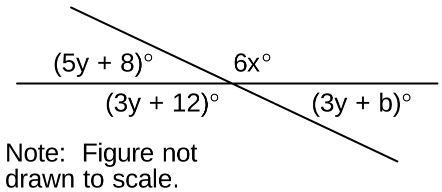
(a) 180 (b) 270 (c) 300 (d) 360
(e) It cannot be determined from the given information.

Example 9: In the given figure, what is the sum of the measures of a, b, c, d, e, f and g?



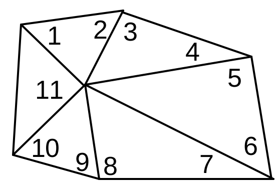
(a) 540 - 2s - 2t
(b) 900
(c) 540
(d) 360
(e) It cannot be determined from the given information.

Example 10: In the given figure, what is the value of x?



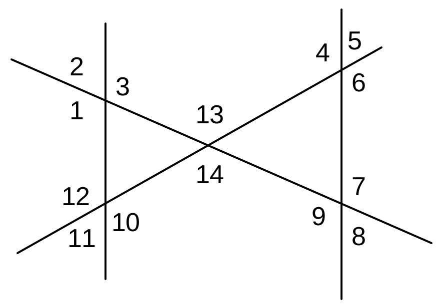
(a) 10
(b) 12
(c) 16
(d) 20
(e) It cannot be determined from the given information.

Example 11: If the m∠11 = 90°, what is the average (arithmetic mean) of the degree measures of angles 1 through 10?



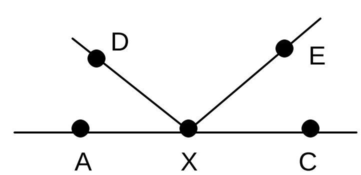
(a) 27 (b) 50 (c) 54
(d) 60 (e) 63

Example 12: In the given figure, what is the sum of the degree measures of ∠1, ∠2, ∠3, ∠4, ∠5, ∠6, ∠7, ∠8, ∠9, ∠10, ∠11, ∠12, ∠13 and ∠14?



(a) 360
(b) 720
(c) 1440
(d) 1800
(e) It cannot be determined from the given information.

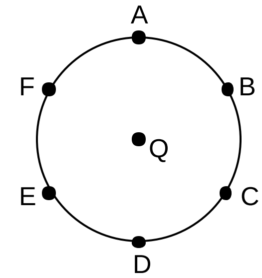
Example 13: In the given figure, if m∠AXE = 126° and m∠CXD = 135°, what is the measure of ∠DXE?



(a) 77 (b) 78 (c) 79
(d) 80 (e) 81

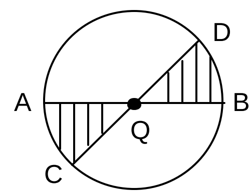
Example 14: In circle Q, points A, B, C, D, E and F divide the circle into 6 equal arcs. If the area of the circle is 36π, what is the length of arc BD?

(a) 2π (b) 3π (c) 4π (d) 6π (e) 12π



Example 15: Circle Q has a radius of 6. If the total area of the shaded region is 6π, what is the measure of $\angle AQD$?

(a) 120° (b) 135° (c) 144° (d) 150° (e) 165°

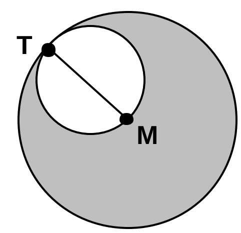


Example 16: The centers of two circles are 7 cm apart. The area of one circle is 100π, and the circumference of the other circle is 6π. At how many points do the circles intersect?

(a) 0 (b) 1 (c) 2 (d) infinite
(e) It cannot be determined from the given information.

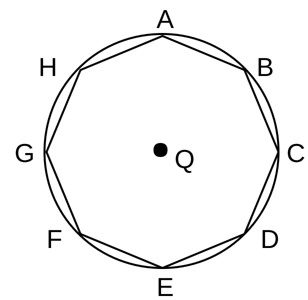
Example 17: In the given figure, M is the center of the large circle, and $\overline{TM}$ is a diameter of the small circle. What is the ratio of the area of the small circle to the area of the shaded region?

(a) 1:2 (b) 1:3 (c) 1:4 (d) 2:3

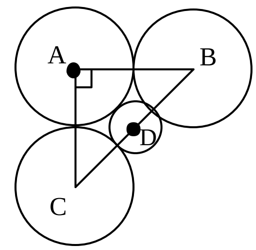


Example 18: In the given figure, regular octagon ABCDEFGH is inscribed in circle Q. If circle Q has a radius of r, which of the following represents the length of arc GH?

(a) $\frac{\pi r}{4}$ (b) $\frac{\pi r}{8}$ (c) $\frac{\pi r^2}{2}$ (d) $\frac{\pi r^2}{4}$ (e) $\frac{\pi r^2}{8}$



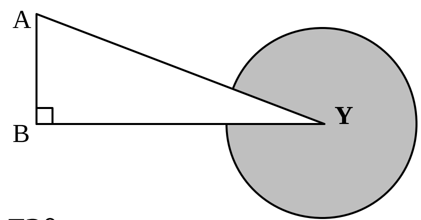
Example 19: In the given figure, circles A, B and C each have a radius of 1. What is the radius of circle D?



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\sqrt{2} - 1$

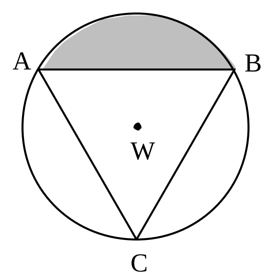
(d) $\sqrt{2} - 2$ (e) $\frac{\sqrt{2}}{3}$

Example 20: In the given figure, circle Y has a radius of 10. If the area of the shaded region is 90π, what is the measurement of $\angle A$?



(a) 10° (b) 36° (c) 54° (d) 60° (e) 72°

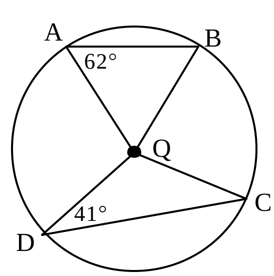
Example 21: Δ ABC is equilateral and is inscribed in circle W, which has a radius of 6. What is the perimeter of the shaded region?



(a) $6 + 2\pi$ (b) $6\sqrt{2} + 2\pi$ (c) $6\sqrt{2} + 4\pi$

(d) $6\sqrt{3} + 2\pi$ (e) $6\sqrt{3} + 4\pi$

Example 22: In circle Q, $m\angle A = 62°$ and $m\angle D = 41°$. What is the $m\angle AQD + m\angle BQC$?

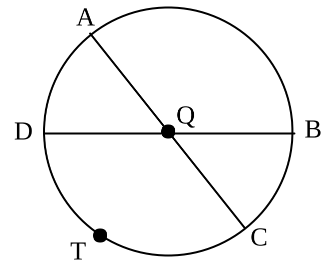


(a) 72° (b) 98° (c) 124°

(d) 196° (e) 206°

Example 23: In circle Q, $\overline{AC}$ and $\overline{DB}$ are diameters of length 10. If $m\angle AQD = 72°$, what is the length of arc DTC?

(A) 2π (B) 3π (C) 4π (D) 6π (E) 8π



Example 24: If the circumference of circle X is 4π cm longer than the circumference of circle Y, then how many cm longer is the radius of circle X than the radius of circle Y?

(a) 2π (b) 4π (c) 2 (d) 4
(e) It cannot be determined from the given information.

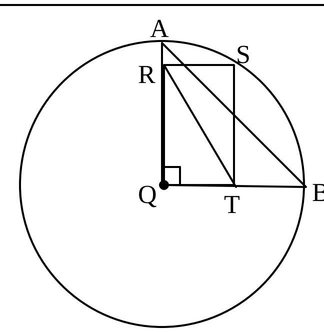
Example 25: A wheel comes off a race car and rolls down the track 400 feet. If the radius of the wheel is 2 feet, which of the following is the closest approximation of the number of revolutions that the wheel makes while traveling this 400 feet?

(a) 30 (b) 60 (c) 100 (d) 200 (e) 400

Example 26: The circumference of circle Q is 20π cm. The sum of the length and the width of rectangle RSTQ is 13 cm. What is the perimeter of quadrilateral RABT in centimeters?

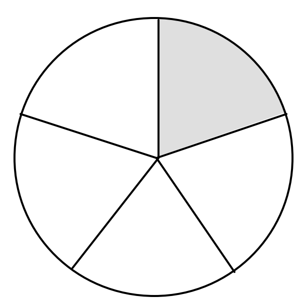
(a) $10 + 10\sqrt{2}$ (b) $20 + 20\sqrt{2}$ (c) $13 + 10\sqrt{2}$

(d) $17 + 10\sqrt{2}$ (e) $17 + 20\sqrt{2}$



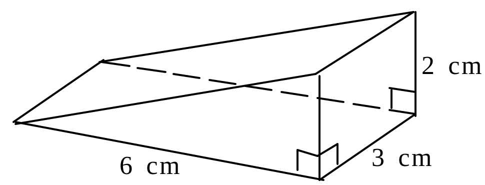
Example 27: This circle has an area of 625π sq cm and is divided into five equal regions. What is the perimeter of the shaded region in centimeters?

(a) $5\pi + 50$ (b) $10\pi + 50$ (c) $25\pi + 50$
(d) $10\pi + 20$ (e) $125\pi + 50$



Area and volume

Example 28: The given figure is half of a rectangular solid. Find the surface area of the figure in square centimeters.



(a) $6\sqrt{5} + 36$ (b) $6\sqrt{10} + 12$
(c) $6\sqrt{10} + 30$ (d) $6\sqrt{10} + 36$
(e) $6\sqrt{10} + 48$

Example 29: Find the ratio of the area of a rectangle whose length is three times its width to the area of an isosceles right triangle whose legs are each equal to the width of the rectangle.

(a) $\frac{6}{1}$ (b) $\frac{3}{1}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{9}{1}$

Example 30: Find the ratio of the area of a rectangle whose length is three times the width of the area of an isosceles right triangle whose hypotenuse is equal to the width of the rectangle.

(a) $\frac{12}{1}$ (b) $\frac{6}{1}$ (c) $\frac{3}{1}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$

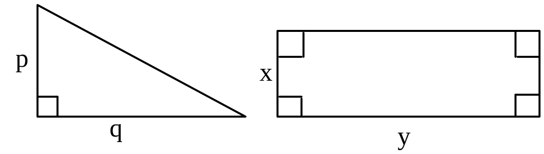
Example 31: The volume of a cube is 64 cubic centimeters. What is the total surface area in centimeters?

(a) 16 (b) 64 (c) 96 (d) 256 (e) 384

Example 32: The lengths of the edges of a rectangular solid are all whole numbers. If the volume of the solid is 13 cubic centimeters, what is the total surface area in square centimeters?

(a) 28 (b) 52 (c) 54 (d) 78 (e) 169

Example 33: If the areas of these two figures are equal and (p)(q) = 100, what is the value of (x)(y)?



(a) 10 (b) 20 (c) 50
(d) 100 (e) 200

Example 34: A tank that has a height of 8 feet and a base that is 2 feet by 2 feet contains 10 cubic feet of water. If 2 cubic feet of water are added to the tank, how many inches will the water in the tank rise?

(a) 2 (b) 6 (c) 12 (d) 24 (e) 1/2

Example 35: A rectangular floor, 10 feet by 12 feet, is going to be tiled with square tiles that each have a perimeter of one foot. What is the least number of tiles needed to cover the floor?

(a) 22 (b) 120 (c) 480 (d) 1200 (e) 1920

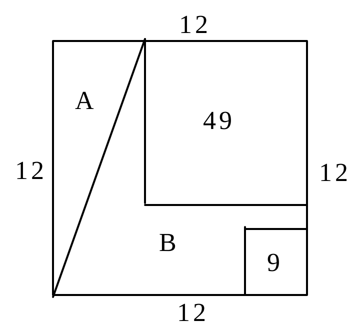
Example 36: The volume of a cube with an edge of $\sqrt{3}$ is how many times the volume of a cube with an edge $\sqrt[3]{3}$?

(a) 3 (b) $\sqrt[3]{3}$ (c) $\sqrt{3}$ (d) $3\sqrt{3}$ (e) 9

Example 37: The height of Δ ABC is 80% of the width of rectangle RSTQ. The base of Δ ABC is 75% of the length of rectangle RSTQ. If the area of Δ ABC is 12, what is the area of rectangle RSTQ?

(a) 9 (b) 20 (c) 30 (d) 40 (e) 80

Example 38: A square with each side equal to 12 is divided into 4 regions. One is a square with an area of 49; another is a triangle named "A" and another is a polygon named "B." What is the area of polygon "B"?



(a) 26 (b) 50 (c) 56 (d) 86
(e) It cannot be determined from the given information.

Example 39: Five cubic blocks are stacked on top of each other. The block on the bottom has an edge of 3 1/2 feet. Each additional block that is stacked has an edge that is 1/2 foot less than the preceding block. What is the ratio of the volume of the block on top to the volume of the block on the bottom?

(a) $\frac{1}{8}$ (b) $\frac{1}{5}$ (c) $\frac{9}{49}$ (d) $\frac{3}{7}$ (e) $\frac{27}{343}$

Example 40: What is the volume of a cube with a surface area of $96s^2$?

(a) $16s^3$ (b) $64s^3$ (c) $256s^3$ (d) $1024s^3$ (e) $9216s^3$

Example 41: If the area of a square is equal to the area of an isosceles right triangle, what is the ratio of a side of the square to the hypotenuse of the right triangle?

(a) $\frac{1}{1}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}}{1}$ (e) $\frac{2}{3}$

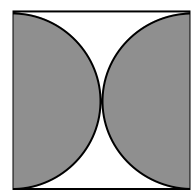
Example 42: What is the area of an equilateral triangle in which each side is equal to 1/2?

(a) $\frac{\sqrt{3}}{64}$ (b) $\frac{\sqrt{3}}{16}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{\sqrt{3}}{2}$ (e) 3

Example 43: What is the ratio of an edge of a cube with a volume of 135 to an edge of a cube with a volume of 40?

(a) 3:1 (b) 3:2 (c) 27:8 (d) 2:3 (e) 8:27

Example 44: The given diagram is a square with two semicircles. What is the ratio of the shaded area to the area of the square?



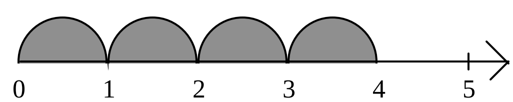
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$ (e) $\frac{\pi}{16}$

Example 45: On a map, 1 inch represents 6 miles. A circle on the map with a circumference of 4π inches represents a circular region with what area?

(a) 8π miles (b) 24π miles (c) 144π miles

(d) 288 miles (e) 576 miles

Example 46: Four semicircles are drawn on the number line. What is the total shaded area?

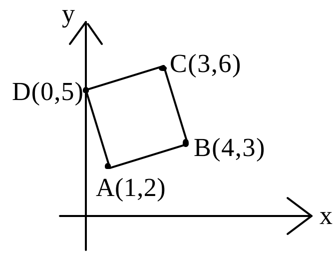


(a) $\frac{1}{4}\pi$ (b) $\frac{1}{2}\pi$ (c) π (d) 2π (e) 4π

The coordinate plane

Example 47: ABCD is a square. What is the area of ABCD?

(a) 9 (b) 10 (c) 12 (d) 14 (e) 16



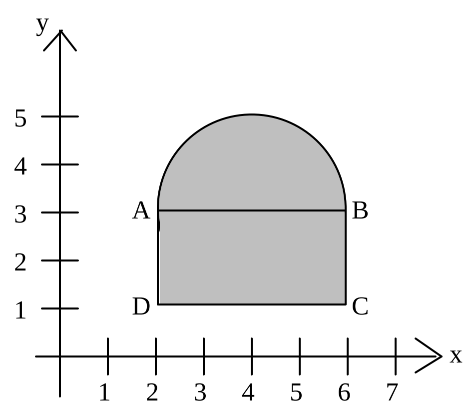
Example 48: What is the area of a triangular region that is bounded by the x-axis, the y-axis and the graph of line x + y = 5?

(a) 5 (b) 10 (c) $12\frac{1}{2}$ (d) 25 (e) $5\sqrt{2}$

Example 49: If $\overline{AB}$ is a diameter parallel to the x-axis and quadrilateral ABCD is a rectangle, what is the perimeter of the shaded region?

(a) $2\pi + 8$ (b) $4\pi + 8$ (c) $2\pi + 10$

(d) $4\pi + 4$ (e) $4\pi + 10$



Answers

1. d	8. d	15. d	22. e	29. a	36. c	43. b
2. a	9. c	16. b	23. b	30. a	37. d	44. c
3. c	10. b	17. b	24. c	31. c	38. c	45. c
4. b	11. e	18. a	25. a	32. c	39. e	46. b
5. c	12. c	19. c	26. d	33. c	40. b	47. b
6. c	13. e	20. c	27. b	34. b	41. b	48. c
7. c	14. c	21. e	28. d	35. e	42. b	49. a

Math concept #9

A hodgepodge of tough ones

Example 1: Joe's car goes 5/6 as fast as Mike's car. Sue's car goes 4/5 as fast as Joe's car. Mike's car goes how many times as fast as the average of the other two cars' speeds?

(a) $\frac{2}{3}$ (b) $\frac{9}{11}$ (c) $\frac{60}{49}$ (d) $\frac{4}{3}$ (e) $\frac{3}{2}$

If you need a hint, look at the bottom of this page.

Example 2: At our cafeteria, P pints of milk are needed per month for each student. At this rate, T pints of milk will supply S students for how many months?

(a) PTS (b) $\frac{T}{PS}$ (c) $\frac{PT}{S}$ (d) $\frac{PS}{T}$ (e) $\frac{S}{PT}$

Example 3: If a + b = c + d, what is the average of a, b, c and d in terms of c and d?

(a) c + d (b) $\frac{c+d}{2}$ (c) $\frac{c+d}{4}$ (d) $\frac{c+d}{8}$ (e) $\frac{cd}{2}$

Example 4: The sum of two consecutive odd integers is S. In terms of S, what is the value of the larger of these integers?

(a) $\frac{S}{2}$ (b) $\frac{S}{2} + 2$ (c) $\frac{S}{2} + 3$ (d) $\frac{S+1}{2}$ (e) $\frac{S+2}{2}$

Example 5: The stock market went up 20% Monday and from that point decreased 25% on Tuesday. What percent of the closing Tuesday price will the market have to rise on Wednesday in order to be exactly where it started on Monday?

(a) 5% (b) 9% (c) $9\frac{1}{9}$% (d) 10% (e) $11\frac{1}{9}$%

Answers are at the bottom of the last page.

Hint for Example 1: Call Mike's speed 12 and go from there. Joe's speed will be 10, and Sue's will be 8.

Example 6: If $xy = -3$ and $(x + y)^2 = 15$, what is the value of $x^2 + y^2$?

(a) 6 (b) 12 (c) 15 (d) 21 (e) 225

<u>I hope that you remember the rule.</u>

Whenever you see any of these six expressions, you immediately write the expression that it is equal to. Do this even before you try to figure what the problem is asking for.

$(x + y)^2$	$(x - y)^2$	$(x + y)(x - y)$
$x^2 + 2xy + y^2$	$x^2 - 2xy + y^2$	$x^2 - y^2$

So, if you see $(x + y)^2$, you write $x^2 + 2xy + y^2$.

If you see $x^2 - y^2$, you write $(x + y)(x - y)$.

etc.

Example 7: The height of a triangle is 50% of the width of a rectangle. The base of the triangle is 30% of the length of the rectangle. If the area of the rectangle is 80, what is the area of the triangle?

(a) 6 (b) 12 (c) 24 (d) 40 (e) 80

Example 8: Thirty students took the SAT. As you know, it is scored from 200 to 800. Exactly 20 of the students made a score greater than or equal to 500. What is the lowest possible average of all thirty scores?

(a) 200 (b) 300 (c) 400 (d) 500 (e) none of the above

Example 9: Forty students took an algebra test, and the average score was Q. Scores on the test ranged from 0 to 94 inclusive. If the average of the first 30 tests that were graded was 70, what is the difference between the greatest possible value of Q and the least possible value of Q?

(a) $23\frac{1}{2}$ (b) $24\frac{1}{2}$ (c) 47 (d) $52\frac{1}{2}$ (e) 76

Example 10: If X is 20% of Y, then Y is what percent of X?

(a) 20% (b) 50% (c) 80% (d) 400% (e) 500%

Example 11: If $(x + y)^2 = x^2 + y^2$, which of the following statements must be true?

(a) $x = y$ (b) If $x = 0$, then $y = 0$. (c) If $x > 0$, then $y < 0$.
(d) If $x > 0$, then $y = 0$. (e) $x \neq y$

Example 12: On the mythical planet Typos, it takes 3 parts of inon and 2 parts of flit to make octos. It takes 5 parts of lactoc and one part of flit to make mylenol. If an amount of octos is mixed with an equal amount of mylenol, what part of the mixture is flit?

(a) $\frac{3}{11}$ (b) $\frac{1}{4}$ (c) $\frac{1}{15}$ (d) $\frac{17}{30}$ (e) $\frac{17}{60}$

Example 13: The length of rectangle X is 20% longer than the length of rectangle Y. The width of rectangle X is 30% shorter than the width of rectangle Y. What is the ratio of the area of rect. X to rect. Y?

(a) 1:1 (b) 4:5 (c) 5:4 (d) 3:5 (e) 21:25

Example 14: Solve the following equation for x. $x + 2y = x(6z + 1)$

(a) $x = 6z - 2y + 1$ (b) $x = \frac{y}{3z}$ (c) $x = \frac{y}{6z + 1}$ (d) $x = \frac{2y - 1}{6z + 1}$
(e) none of the above

Example 15: A class of 20 students had an average score of 71 on a test. Another class that had 25 students had an average score of 80 on the test. What was the average score on the test of all 45 students?

(a) 75.5 (b) 76 (c) 76.5 (d) 77 (e) 80

Example 16: What is the value of (24% of x) - $\frac{6x}{25}$?

(a) $\frac{18}{25}$ (b) $23\frac{19}{25}x$ (c) $0.48x$ (d) $\frac{18}{25}x$ (e) 0

Example 17: If $x = \sqrt{15}$ and $y = \sqrt{18}$, what is the value of $(x - y)^2 + (x + y)^2$?

(a) 33 (b) 66 (c) 112 (d) 1098 (e) 0

Example 18: If $-1 < x < 0$, then which of the following is true?

(a) $x^2 < x^3 < x$ (b) $x < x^2 < x^3$ (c) $x^2 < x < x^3$ (d) $x < x^3 < x^2$
(e) $x^3 < x^2 < x$

Example 19: If each of the following 6 numbers (20, 19, 21, 15, 14, 17) were increased by x, the average of the resulting 6 numbers would be 19. What is the value of x?

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) 1 (e) $\frac{4}{3}$

Example 20: If the average of 5 numbers is -10 and the sum of two of the numbers is 16, what is the average of the other three numbers?

(a) $\frac{-34}{3}$ (b) -34 (c) -22 (d) $\frac{-66}{5}$ (e) -66

If you have traveled this far and done all the problems, keep going. The end is near.

Answers 1. d 2. b 3. b 4. e 5. e 6. d 7. a 8. c 9. a 10. e 11. d
12. e 13. e 14. b 15. b 16. e 17. b 18. d 19. e 20. c

Math concept #10 (the last)

Tying up a few loose ends
Assorted problems

Example 1: A grocer sold 1/3 of his milk on Monday and 2/5 of what was left on Tuesday. If he had 300 gallons after Tuesday, how many gallons of milk did the grocer start with?

(a) 250 (b) 300 (c) 500 (d) 600 (e) 750

Start by calling the number of gallons that the grocer started with x.

Then write an equation and solve. It would be $x - \frac{1}{3}x - \frac{2}{5}(x - \frac{1}{3}x) = 300$.

Example 2: This problem is similar to Example 1. See if you can write an equation and solve it.

A boy, who is taking a girl to the fair, spends 1/4 of the total money that he takes at the football throw booth trying to win his date a stuffed lion that is 5 feet high. He spends 2/3 of the remaining money on rides. If he then has $10 left to buy food and drinks, how much money did he originally take to the fair?

(a) $10 (b) $30 (c) $40 (d) $50 (e) $60

Example 3: This next problem is a little different and a little easier. A grocer has 400 gallons of milk. He sells 2/5 of it on Monday and 3/4 of what is left on Tuesday. How many gallons does the grocer have left after Tuesday?

(a) 30 (b) 60 (c) 80 (d) 120 (e) 270

Note: You might try to solve #1 and #2 in the same manner that you solved #3, by testing each answer choice to see if it works. As an example, review Example 2.

Start with $60. If that was what the boy started with, and he spent 1/4 of it at the football booth, he would have $45 left. If he then spent 3/4 of that on rides, he would have $11.25 left. Since $10 is the answer, (e) $60 is not the correct choice.

Now test (d) $50. This will not work either.

Now test (c) $40. If the boy spent 1/4 of this at the football booth, he would have $30 left. If he spent 2/3 of this on rides, he would have $10 left. So (c) $40 is the correct choice. Since this method is very time-consuming, do not attempt this unless you have plenty of time.

Example 4: A grocer sold 1/2 of his stock of milk on Monday. On Tuesday he sold 40 gallons, and at that point he had 3/10 of his original stock left. How many gallons of milk were in his original stock?

(a) 100 (b) 150 (c) 180 (d) 200 (e) 220

Example 5: *Try to do this one without multiple choices. The answer is at the bottom of the last page of this section.*
A grocer sold 1/2 of his stock of milk on Monday. On Tuesday he sold 42 gallons, and at that point he had 2/7 of his original stock left.
How many gallons of milk were in his original stock?

Example 6: Last month Joe spent 1/4 of his paycheck on his house payment and 2/5 of the remainder on food. If he then had $540 left over, what was the original amount of his paycheck?

(a) $900 (b) $1000 (c) $1100 (d) $1200 (e) $2000

Example 7: There are 60 people in a room. 2/3 are under 21 years of age, and 2/5 are male. What is the maximum number of <u>females</u> in the room who can be under 21?
(a) 16 (b) 24 (c) 30 (d) 36 (e) 40

(Remember that an underlined word means, "Watch out! You are about to make a dumb mistake." So be careful.)

First figure how many people are under 21 years of age. You should get 40. Then figure how many of those in the room are male. You should get 24. Now, figure how many of those people in the room are female. You should get 36. Now ask yourself if it is possible for all 36 of these females to be under 21.

Example 8: Use the information in #7 to answer the following question:
What is the minimum number of females in the room who must be under 21?
(a) 8 (b) 16 (c) 24 (d) 36 (e) 40

Example 9: Thirty people were sitting on a row at the Cotton Bowl for the Texas-Oklahoma game. Exactly 15 had some orange clothes on, exactly 12 had some red clothes on, and exactly 5 had both orange and red clothes on. What is the total number of people sitting on the row that had neither red nor orange clothes on?

(a) 2 (b) 3 (c) 8 (d) 10 (e) 15

Example 10: At Washington High School, 4/5 of the teachers are married, and 4/5 of those who are married have children. What fractional part of the teachers are married but do not have children?

(a) $\frac{1}{25}$ (b) $\frac{4}{25}$ (c) $\frac{1}{5}$ (d) $\frac{9}{25}$ (e) $\frac{2}{5}$

Example 11: The senior class has 90 students, and 2/3 of the class are girls. 2/3 of the girls in the class are 18 years of age or older. What fractional part of the senior class is girls <u>younger</u> than 18?

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{4}{9}$

Example 12: At Washington High School, the baseball team has 18 members, and the basketball team has 15 members. If a total of 17 students participate in only one of the two sports, how many students participate in both sports?
(a) 3 (b) 7 (c) 8 (d) 15 (e) 16

Look at it like this. The sum of the number of names on the two rosters is 33. Seventeen of these names appear only once. This leaves sixteen that are composed of pairs of names, representing players that play on both teams. How many pairs are there in 16?

Example 13: At Washington High School, there are 28 students in the math club and 25 students in Key Club. 31 students are members of just one of these two clubs. How many students are members of both clubs?
(a) 3 (b) 6 (c) 8 (d) 11 (e) 22

Example 14: (a trick question just to keep you on guard ...) In a class of 200 seniors, 100 are taking calculus but not physics, and 60 are taking physics but not calculus. How many seniors are taking both physics and calculus?
(a) 20 (b) 40 (c) 60 (d) 100 (e) One cannot tell for sure.

Further discussion of Example 14 The answer could be as few as zero (why would any senior have to be taking both courses?) or as many as 40. Since 100 are taking calculus but not physics, and 60 are taking physics but not calculus, that leaves 40 seniors whose schedules are unknown except that none of the 40 is only taking one of the two courses. They either would be taking neither or both.

<u>The following problems deal with time.</u>

Example 15: If it is now 5 p.m. Monday, 251 hours from now, what time and day will it be?

(a) 4 a.m. Monday (b) 1 a.m. Tuesday (c) 4 a.m. Thursday
(d) 1 a.m. Friday (e) 4 a.m. Friday

Example 16: One-fourth of an hour is what fraction of the time between 1 a.m. Monday and 2 p.m. Tuesday of the same week?

(a) $\frac{1}{25}$ (b) $\frac{1}{100}$ (c) $\frac{1}{37}$ (d) $\frac{1}{74}$ (e) $\frac{1}{148}$

Example 17: The second hand of a clock rotates how many times faster than the hour hand?

(a) 600 (b) 720 (c) 1440 (d) 3600 (e) 7200

Example 18: When it is 7 p.m. in San Francisco, it is 10 p.m. in New York. If a plane leaves San Francisco at 9:30 p.m. San Francisco time and arrives in New York at 4:30 a.m. New York time, how many hours did the flight take?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

<u>The following concern counting. (They are harder than you might think.)</u>

Example 19: If all students in seats 120 to 178 left the football game, how many students left?

(a) 57 (b) 58 (c) 59 (d) 60 (e) 61

Example 20: For an algebra assignment, a student had to do all even problems from 20 through 100. How many problems were assigned?

(a) 80 (b) 81 (c) 39 (d) 40 (e) 41

Answers: 1. e 2. c 3. b 4. d 5. 196 gallons 6. d 7. d 8. b 9. c 10. b 11. b
12. c 13. d 14. e 15. e 16. e 17. b 18.b 19. c 20. e

The Big Review

The following is a review of many of the difficult problems from the concept worksheets.
The answers are given after problem #10, and then detailed explanations of each problem follow.

1. Joe has savings of s dollars in a sock in his drawer. If he earns x dollars per week and spends y dollars per week, how many weeks will his savings last if $y > x$?

(a) $\frac{s}{x - y}$ (b) $\frac{y - x}{s}$ (c) $\frac{s - y}{x}$ (d) $\frac{x - y}{s}$ (e) $\frac{s}{y - x}$

2. On a geometry test, the average score for the class was 90. If 20% scored 100, and 30% scored 80, what was the average score for the remainder of the class?

(a) 86 (b) 88 (c) 90 (d) 92 (e) 94

3. The original price of a car is raised by 30% on Monday. That new price is then lowered by 30% on Tuesday and sold. The selling price is what percent of the original price?

(a) 0% (b) 100% (c) 30% (d) 90% (e) 91%

4. If $x^2 + y^2 = 20$ and $xy = -4$, what is the value of $(x + y)^2$?

(a) 20 (b) 16 (c) 12 (d) 392 (e) 256

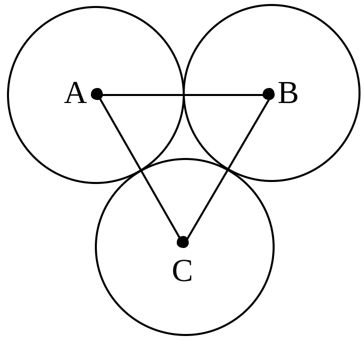
5. If the perimeter of a square is 36, what is the length of the diagonal of the square?

(a) 6 (b) $6\sqrt{2}$ (c) 9 (d) $9\sqrt{2}$ (e) 12

6. The total cost of r roses is c cents.
What is the cost of 3 roses in <u>dollars</u>?

(a) $\frac{300r}{c}$ (b) $\frac{3c}{r}$ (c) $\frac{300c}{r}$ (d) $\frac{3c}{100r}$ (e) 3cr

7. Circles A, B and C are congruent to each other. The sum of the circumferences of the three circles is 36π. What is the area of ΔABC?



(a) 36 (b) 72 (c) $18\sqrt{3}$ (d) $36\sqrt{3}$ (e) 12π

8. For all real numbers, let *X be the smallest number that can be added to X to get the smallest integer greater than X. For example, *2.7 = 0.3 because 2.7 + 0.3 will equal 3, which is the smallest integer greater than 2.7. What is the value of $(1.6 - {*1.6})^2$?

(a) 0 (b) 2.56 (c) 1.44 (d) 1 (e) 0.144

9. $\frac{1}{2^5} - \frac{1}{2^6}$ is equal to which of the following?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2^6}$ (d) $-\frac{1}{2^6}$ (e) $\frac{1}{2^5}$

10. A florist was not able to sell 20 of the mums that he made for a homecoming game. Monday he sold 1/4 of the mums that he started with, and Tuesday he sold 2/3 of those remaining after Monday. How many mums did he originally have to sell?

(a) 40 (b) 60 (c) 80 (d) 100 (e) 120

Answers: 1. e 2. d 3. e 4. c 5. d 6. d 7. d 8. c 9. c 10. c

Explanations

1. An easy method to solve this problem is to:
   1. put in your values
   2. solve your problem
   3. put your values into the answer choices and find which choice will result in your answer.

   Start by writing something like the following:

   let s = $100
   let x = $10
   let y = $15

   The answer to this made-up problem is 20 weeks.
   If Joe spends $5 more than he takes in each week, it will take him 20 weeks to spend the entire $100 savings.

   Now, put your values (s = 100, x = 10 and y = 15) into each of the answer choices and see which choice gives 20 weeks.

Note: Remember to start with choice (e) if you need to test all choices.

In this problem, (e) is the correct answer. $\frac{100}{15 - 10} = \frac{100}{5} = 20$

2. This is a "total stuff" problem.
   1. Decide how many people you want to take the test.
   2. Ten would be a good number.
   3. The "total stuff" is the total number of points that the ten received on the test to get an average of 90. (90 X 10 = **900**)
   4. 20%, or two of these 10, made a 100. This takes up 200 of the total stuff.
   5. 30%, or three of these 10, made an 80. This takes up 240 of the total stuff.
   6. So, a total of 440 of the 900 points are taken, leaving 460 points for the other 50%, which is 5 people.
   7. Divide 460 by 5, and their average score will be 92, choice (d).

If you have a difficult averaging problem, find the "total stuff."

3. If you are ever asked for a ratio or a percent of one thing to another, write out with phrases what is divided by what.

1. In this problem write:

$$\frac{\text{final selling price}}{\text{original selling price}}$$

2. Call the original price of the car \$100. (Note that 100 is a good number to use when dealing with percents.)
3. An increase of 30% would be an increase of \$30.
4. The new price of the car would be \$130.
5. If that price is decreased by 30%, then the selling price is 91. (30% • \$130 = \$39 and \$130 - \$39 = \$91)
6. Fill in the following ratio:

$$\frac{\text{final selling price}}{\text{original selling price}} = \frac{\$91}{\$100} = .91 = \mathbf{91\%}$$

4. For one last time, learn these common factors.

<u>I hope that you remember the rule.</u>

Whenever you see any of these six expressions, you immediately write the expression that it is equal to. Do this even before you try to figure what the problem is asking for.

$\boxed{(x + y)^2}$	$\boxed{(x - y)^2}$	$\boxed{(x + y)(x - y)}$
$\boxed{x^2 + 2xy + y^2}$	$\boxed{x^2 - 2xy + y^2}$	$\boxed{x^2 - y^2}$

So, if you see $(x + y)^2$, you write $x^2 + 2xy + y^2$.

If you see $x^2 - y^2$, you write $(x + y)(x - y)$. etc.

In problem #4 you:

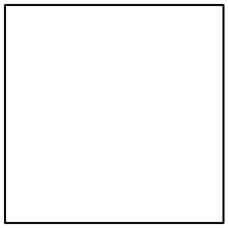
1. Immediately write $x^2 + 2xy + y^2$,

2. rearrange the three terms to get $(x^2 + y^2) + (2xy)$,

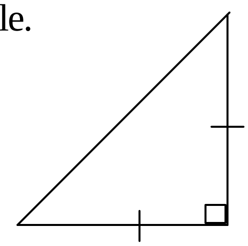
3. substitute and get $(20) + (2 \bullet -4) = 20 - 8 = \mathbf{12}$.

5. Please draw diagrams for geometry problems.

1. For #5, draw a square.



2. You can find a diagonal if you know a side because the diagonal and two sides will form a 45°–45°–90° triangle.



3. The diagonal will be equal to (a side)($\sqrt{2}$).
4. Since the perimeter is 36, a side will be 9, and the answer will be **(9 $\sqrt{2}$)**.

6. The total cost of r roses is c cents.
What is the cost of 3 roses in dollars?

An easy way to do this problem and ones like it is to insert your own values, find an answer for your problem, and then see which of the answer choices will give the same answer when your values for the variables are inserted.

1. Let r be 1.
2. Let c be 200.
3. So, 3 roses would cost 600 cents or **6** dollars.
4. Insert 1 for r and 200 for c in the answer choices and see which will result in the answer that you got in your problem, **6**.
5. Remember that you should start with (e) if you need to test all of the answer choices.

Choice (d) will give your correct answer of 6.

$$\frac{3c}{100r} = \frac{3(200)}{100(1)} = \mathbf{6}$$

7. 1. Since the sum of circumferences is 36π, each circle's circumference will be 12π.

2. To find the radius of a circle when the circumference is known, use the formula $c = 2\pi r$.

3. Insert 12π for c and get $12\pi = 2\pi r$.

4. Solve for r and get $6 = r$.

5. Notice that a side of the Δ is made up of two radii.

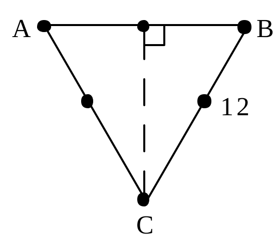
6. Therefore, a side of the triangle would be 12. This will cause the triangle to be equilateral.

7. If you remember that the area of an equilateral triangle is found by the formula $a = \frac{s^2\sqrt{3}}{4}$,

then you can insert 12 for the s and get $a = \frac{12^2\sqrt{3}}{4} = 36\sqrt{3}$.

8. If you do not remember this formula, then you need to use $a = \frac{1}{2}bh$.

9. Draw in an altitude from point C to side $\overline{AB}$. This is h.



10. Since the triangle is equilateral, $m\angle B = 60°$.

11. Since BC = 12, the altitude of the triangle will be $\mathbf{6\sqrt{3}}$.

You get this if you know that the side opposite the $60°\angle = \frac{\text{hypotenuse}\sqrt{3}}{2}$

12. Now insert the base and the altitude into the formula $a = \frac{1}{2}bh$. $a = \frac{1}{2}(12)(6\sqrt{3}) = \mathbf{36\sqrt{3}}$.

8. To find *1.6

1. Find the smallest integer that is greater than 1.6, which is **2**.

2. What must be added to 1.6 to get 2? **0.4**

3. Now the problem is $(1.6 - 0.4)^2$.

4. This will be $(1.2)^2$, which is equal to **1.44.**

9. Remember to factor, factor, factor, factor, factor, factor.

$$\frac{1}{2^5} - \frac{1}{2^6} = \frac{1}{2^5}\left(1 - \frac{1}{2^1}\right) =$$

$$\frac{1}{2^5}\left(\frac{1}{2}\right) = \frac{1}{2^6}$$

10. If this problem is done with algebra, find expressions that would fit into the following sentence.

(Original amount of flowers) minus (1/4 that amount) minus (2/3 of what is left over) = 20.

Call the original amount x. The equation expressing the above sentence is

$$x = \frac{1}{4}x - \frac{2}{3}\left(x - \frac{1}{4}x\right) = 20$$

$$\frac{3}{4}x - \frac{2}{3}x + \frac{2}{12}x = 20$$

$$\frac{9}{12}x - \frac{8}{12}x + \frac{2}{12}x = 20$$

$$\frac{3}{12}x = 20 \qquad \frac{1}{4}x = 20$$

So, x = 80.

Another method: If the florist had 20 at the end after he had sold 2/3 on Tuesday, then the 20 would represent 1/3 of what he had at the beginning of Tuesday. If this 60 was what was left over after he sold 1/4 of what he started out with, then he must have started with **80.**